Martingale Optimal Transport

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(Dated: October 23, 2014)

The original transport problem is to optimally move a pile of soil to an excavation. Mathematically, given two measures of equal mass, we look for an optimal map that takes one measure to the other one and also minimizes a given cost functional. Kantorovich relaxed this problem by considering a measure whose marginals agree with given two measures instead of a bijection. This generalization linearizes the problem. Hence, allows for an easy existence result and enables one to identify its convex dual.

In robust hedging problems, we are also given two measures. Namely, the initial and the final distributions of a stock process. We then construct an optimal connection. In general, however, the cost functional depends on the whole path of this connection and not simply on the terminal value. Hence, one needs to consider processes instead of simply the transport maps. The probability distribution of this process has prescribed marginals at terminal and initial times. Thus, it is in direct analogy with the Kantorovich measure. But, financial considerations restrict the process to be a martingale. Interestingly, the dual also has a financial interpretation as a robust hedging (super-replication) problem.

In this talk, we prove an analogue of Kantorovich duality: the minimal super-replication cost in the robust setting is given as the supremum of the expectations of the contingent claim over all martingale measures with a given marginal at the maturity. This joint work with Yan Dolinsky from Hebrew University.