A general characterization of the mean field limit for stochastic differential games

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Mean field game (MFG) theory generalizes classical models of interacting particle systems by replacing the particles with rational agents, making the theory applicable in economics and other social sciences. Intuitively, (stochastic differential) MFGs are infinite-population analogs of large-population stochastic differential games of a certain symmetric type, and a solution of an MFG is analogous to a Nash equilibrium. Most research so far has addressed the problem of solving the infinite-population model, typically by way of forward-backward systems of PDEs or McKean-Vlasov SDEs, and the solution is then used to construct approximate equilibria for the finite-population games. This talk discusses some new general results in this direction, but more attention is payed to recent progress on a less well-understood problem: Given for each n a Nash equilibrium for the n-player game, in what sense if any do these equilibria converge as n tends to infinity? The answer is somewhat unexpected, and certain forms of randomness can prevail in the limit which are well beyond the scope of the usual notion of MFG solution considered thus far in the literature. By defining a notion of weak MFG solution, it is shown for a large class of models that the set of weak MFG solutions coincides exactly with the set of possible limits (in distribution) of finite-population approximate Nash equilibria.