Homework 8

Due: 13 April 2016

Problem 1
Prove Proposition 3.20 (the chain rule for conditional independence).

Problem 2
For random variables $X$, $X'$ and $Y$, show that

$$(X,Y) \overset{d}{=} (X',Y) \iff P[X \in A|Y] = a.s. P[X' \in A|Y]$$

for any measurable set $A$.

Problem 3
Assume $(X,Y) \overset{d}{=} (X',Y')$, where $X$ is integrable. Show that $E[X|Y] = a.s. f(Y)$, for some measurable mapping $f$, imply $E[X'|Y'] = a.s. f(Y')$.

Problem 4
Suppose $X \perp Y Z$ and $T \perp (X,Y,Z)$. Show

$$X \perp Y T Z \quad \text{and} \quad X \perp Y (Z,T) .$$

Problem 5*
Let $X$, $Y$ and $Z$ be random variables, and assume that $Y$ is $\sigma(Z)$-measurable. Show that

$$(X,Y) \overset{d}{=} (X,Z)$$

implies $X \perp Y Z$.

Hint: Show first that $P[X \in A|Y] = P[X \in A|Z]$. Then turn the equality in distribution into an almost sure equality.