OPTIMAL INVESTMENT FOR ALL TIME HORIZONS AND EVOLUTION EQUATIONS WITH A WRONG TIME DIRECTION

SERGEY NADTOCHIY

ABSTRACT. I will start by reviewing the existing results in the study of forward performance processes, which serve as the optimality criteria for investment problems with multiple time horizons. I, then, show that the existing definition of a forward performance lacks an important component. Having added the missing part of the definition, I develop an axiomatic justification for this theory. In this modified setting, under the additional Markovian assumption on the market, the optimality criterion, as well as the solution to the associated optimization problem, is characterized by the Hamilton-Jacobi-Bellman equation on a semi-finite time interval. The main difficulty in analyzing the latter equation stems from the fact that it "has time running in a wrong direction": we need to solve this (backward) parabolic PDE forward in time. Using the existing results on the form of the minimal elements of a Martin boundary of a space-time diffusion, I provide an explicit characterization of all positive solutions to some classes of these (ill-posed) equations. In addition, I show how these results extend the classical Widder's theorem on the positive solutions of a time-reversed heat equation.