Martingale optimal transport and robust hedging

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The martingale optimal transport problem is motivated by model-independent bounds for the pricing and hedging exotic options.

In the simplest one-period model, the dual formulation of the robust superhedging cost differs from the standard optimal transport problem by the presence of a martingale constraint on the set of coupling measures. The one-dimensional Brenier theorem has a natural extension. However, in the present martingale version, the optimal coupling measure is concentrated on a pair of graphs which can be obtained in explicit form. These explicit extremal probability measures are also characterized as the unique left and right monotone martingale transference plans, and induce an optimal solution of the Kantorovitch dual, which coincides with our original robust hedging problem.

By iterating the above construction over n steps, we define a Markov process whose distribution is optimal for the n-periods martingale transport problem corresponding to a convenient class of cost functions. Similarly, the optimal solution of the corresponding robust hedging problem is deduced in explicit form. Finally, by sending the time step to zero, this leads to a continuous-time version of the Brenier theorem in the present martingale context, thus providing a new remarkable example of Peacock, i.e. Processus Croissant pour l'Ordre Convexe. Here again, the corresponding robust hedging strategy is obtained in explicit form.