

On facelifting in mathematical finance

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Super-replication of contingent claims in incomplete markets often involves a 'facelift': the payoff function is replaced by an envelope in an appropriate class and the new, facelifted, payoff is priced using complete-market methods. More generally, in optimal stochastic control theory, the facelift appears in the form of a discontinuity of the value function at the terminal time and typically arises when the control set is unbounded. In that case, the Hamiltonian may take infinite values and the facelift typically consist of replacing the terminal payoff by a smallest finite-Hamiltonian majorant.

When utility-based pricing is used instead of super-replication, no facelift is expected, thanks to the smoothing effect of the utility function. Indeed, the Hamiltonian applied to the terminal payoff is always finite. Yet, there is a facelift as soon as the claim being priced is non-replicable. We show this unexpected fact using control-theoretic methods and relate it to the appearance of finitely-additive dual minimizers in the problem of utility maximization with a random endowment. We also analyze the corresponding Hamilton-Jacobi-Bellman equation and provide necessary analytic conditions on its terminal condition for the absence of a facelift. This is joint work with Kasper Larsen.